ISSN 2229-5518

FREE VIBRATION ANALYSIS OF THIN RECTANGULAR CCSS PLATE USING THE FORMULATED FLEXURAL ELEMENT STIFFNESS MATRIX IN FINITE ELEMENT METHOD.

O. M. Ibearugbulem, L. Anyaogu, M.U. Ndukwu, J.I. Arimanwa, K.O. Njoku

Civil Engineering Department, Federal University of Technology Owerri,

NIGERIA.

ABSTRACT

This study presents free vibration analysis of thin rectangular CCSS plate. The formulated flexural element stiffness matrix using finite element method for a selected shape function was used to determine the fundamental natural frequency of CCSS plate under vibration. This study displayed the finite element continuum of CCSS boundary condition through the formulation of the stiffness matrix K and inertia matrix K_λ. The result of discretization of the plate showed the importance of finite element method as approximate method of analysis. The initial resonating frequency at a grid size of n=1 and aspect ratio α=1 was obtained with the formulated stiffness matrix. The fundamental natural frequency λ for other aspect ratios ranging from 1.1 to 2.0 at an increment of 0.1 and grid size n=3 to n=17 was determined. The fundamental natural frequencies under grid size n=17 closest to the solution was used to compare other approximate methods of Njoku, [4], Sakata et al, [10], and Chakravarty, [1]. It was observed from percentage difference ranging from 0.4620% to 0.6229% that the present study and other approximate methods are close. Therefore, we can draw conclusion that the present approximate method of analysis is acceptable for analyzing plates in vibration.

Keywords: *Vibration, Analysis of Rectangular Thin Pate, Finite Element Method, Stiffness, Fundamental Natural Frequency,*

INTRODUCTION

Structural elements have a wide range of applications in engineering. Thin plates are one of these applications that undergoes a wide range of excitations that result to vibration. The wide use of this kind of structures requires a complete investigation of the dynamic behavior in order to develop accurate and reliable design. Uncontrolled vibration can damage mechanical system (like Plates) and most of the time render it ineffective for use in its entire life. It is therefore recommended that a proper method of analysis should be used to investigate the adequate natural frequency of any vibrating system before use.

Finite element analysis is a good approximate method for analyzing plate structures in vibration. The Finite Element Method (FEM) involves division of a structure into discrete elements interconnected at selected nodes. The deformation of each element is expressed by interpolating polynomials whose coefficient are defined in terms of Degrees of Freedom (DOF) that describes the displacements and slopes of selected nodes on the element. Nikolas, [6]. The fourth order differential equation with the flexural continuum

ISSN 2229-5518

energy approach was applied on the selected shape function by using Finite Element Method. At the central deflection of the plate, the corresponding resonating frequency was obtained by application of CCSS plate boundary condition on the formulated flexural stiffness matrix of the plate.

Free vibration analysis of thin rectangular plates that are clamped on adjacent edges and simply supported on opposite adjacent edge (CCSS) has been analyzed using various approach by many researchers in the past to obtain the resonating natural frequency Okafor and udeh[8], Sakata et al [10],Chakraverty [1], Supun and Seyed [11]. The use of variational calculus of fourth order differential equation by these scholars Njoku [6], Ibearugbulem et al [2], Ibearugbulem et al, [3], Oba et al [8], Ezeh et al [4] to obtain the natural frequency of plate. The use of finite element method to determine vibration frequencies and compare them with Levy type solution. Ramu and Mohanty [9]. The Formulated Flexural Element Stiffness Matrix in Finite Element Method to compute the natural frequency of CCSS plate has not being used by any scholar in the past. The use of other approximate method can be time consuming and complex in obtaining solution. Hence Finite Element Method is a powerful approximate method that has an advantage due to its amenability to computer program for quick and accurate analysis. The main purpose of this present study is to use the Formulated Flexural Stiffness Matrix in Finite Element Method to analyze free vibration of thin rectangular plate clamped on two adjacent edges and simply supported on the opposite adjacent edges (CCSS).

GORVERNING DIFFERENTIAL EQUATION FOR TOTAL STRAIN ENERGY OF PLATE IN VIBRATION.

The governing differential equation for total strain energy of plate in vibration is given in Equation (1);

$$\Pi=\frac{D}{2}\int_0^a\int_0^b\left[\left[\frac{\partial^2 w}{\partial x^2}\right]^2+2\left[\frac{\partial^2 w}{\partial x\partial y}\right]^2+\left[\frac{\partial^2 w}{\partial y^2}\right]^2\partial x\partial y\right]-\frac{m\lambda^2}{2}\int\int w^2\partial x\partial y \;=0 \quad (1)$$

Where D is the flexural rigidity of the plate.

m is the mass per unit area of the thin plate.

w is the displacement of the plate.

λ is the fundamental natural frequency of the thin plate.

The flexural rigidity D of the thin plate is expressed in Equation (2);

$$D=\frac{Eh^3}{12(1-\upsilon^2)} \quad (2)$$

E = Modulus of elasticity

h = Plate thickness

v = Poisson's ratio.

Equation (1) is the main equation for the derivation of the Flexural Element Stiffness Matrix.

FLEXURAL ELEMENT STIFFNESS MATRIX FORMULATION.

Ibearugbulem et al [2] expressed the deflection equation 'w' of a plate in Equation 3;

$$[w]=[N][\psi] \quad (3)$$

Where, w = deflection of plate

$[N]$=deflection shape function

$[\psi]$ = coefficients of the displacements i.e. for rotation and deflection of plate.

$$[w_i]=[N_i][\psi] \quad (4)$$

Let $[w_i]$ = deflection and rotation at the nodes of the thin plate.

$[N_i]$= The square matrix of nodal value displacement profile of plate.

Equation (4) can be re-arranged as expressed in Equation (5);

$$[\psi]=[N_i^{-1}]^T[w_i] \quad (5)$$

We obtain the new expression of deflection w in Equation (6) by substituting Equation (5) into Equation (3)

$$w=[N][N_i^{-1}]^T[w_i] \quad (6)$$

ISSN 2229-5518

Substituting Equation (6) into Equation (1) yields Equation (7)

$$,\prod = \frac{D}{2}\left[\int_0^a\int_0^b\left[[N][N_i^{-1}]^T[w_i]\right]_x^{''}\right]^2 dxdy +$$

$$D\left[\int_0^a\int_0^b\left[[N][N_i^{-1}]^T[w_i]\right]_{xy}^{''}\right]^2 dxdy$$

$$+\frac{D}{2}\left[\int_0^a\int_0^b\left[[N][N_i^{-1}]^T[w_i]\right]_y^{''}\right]^2 dxdy -$$

$$\frac{m\lambda^2}{2}\left[\int\int\left[[N][N_i^{-1}]^T[w_i]\right]^2\right] \quad dxdy \qquad (7)$$

Expand Equation (7) to obtain Equation (8).

$$\frac{D}{2}[N_i^{-1}]^T\,[w_i]^T\ \int_0^a\int_0^b\left[[N]^T[N]\right]_x^{''} dxdy.\ [N_i^{-1}][w_i] +$$

$$D[N_i^{-1}]^T\,[w_i]^T\ \int_0^a\int_0^b\left[[N]^T[N]\right]_{xy}^{''} dxdy.\ [N_i^{-1}][w_i]$$

$$+\frac{D}{2}[N_i^{-1}]^T\,[w_i]^T\ \int_0^a\int_0^b\left[[N]^T[N]\right]_y^{''} dxdy.\ [N_i^{-1}][w_i]$$

$$-\frac{m\lambda^2}{2}[N_i^{-1}]^T\,[w_i]^T\ \int\int\left[[N]^T[N]\right] \quad dxdy.\ [N_i^{-1}][w_i] \qquad (8)$$

Extracting the constants $[N_i^{-1}]^T$ and $[w_i]^T$ and minimizing Equation (8) by differentiating with respect to $[w_i]$ yields Equation (9).

$$\prod = [N_i^{-1}]^T\,D.\int_0^a\int_0^b [[N]^{''T}[N]^{''}]_x\, dxdy.\ [w_i][N_i^{-1}]$$

$$+[N_i^{-1}]^T\,2D.\int_0^a\int_0^b [[N]^{''T}[N]^{''}]_{xy}\, dxdy.\ [w_i][N_i^{-1}]$$

$$+[N_i^{-1}]^T\,D.\int_0^a\int_0^b [[N]^{''T}[N]^{''}]_y\, dxdy.\ [w_i][N_i^{-1}]$$

$$-[N_i^{-1}]^T\,m\lambda^2.\int\int\left[[N]^T[N]\quad\right] \quad dxdy.\ [w_i][N_i^{-1}] \qquad (9)$$

Equation (9) is called the General Flexural Element Stiffness Matrix Equation of thin rectangular plate.

Equation (9) expressed in dimensionless R-Q coordinate system yields Equation (10).

$$\prod = [N_i^{-1}]^T\,\frac{D}{a^4}\int_0^1\int_0^1 [[N]^{''T}[N]^{''}]_R\, dRdQ\ .\ [w_i][N_i^{-1}]$$

$$+[N_i^{-1}]^T\,\frac{2D}{a^4\alpha^2}\int_0^1\int_0^1 [[N]^{''T}[N]^{''}]_{RQ}\, dRdQ\ .\ [w_i][N_i^{-1}]$$

$$+[N_i^{-1}]^T\,\frac{D}{a^4\alpha^4}\int_0^1\int_0^1 [[N]^{''T}[N]^{''}]_Q\, dRdQ\ .\ [w_i][N_i^{-1}]$$

$$-[N_i^{-1}]^T\,m\lambda^2\ \int\int\left[[N]^T[N]\quad\right] \quad dRdQ\ .\ [w_i][N_i^{-1}] \qquad (10)$$

Equation (10) is called the General Flexural Element Stiffness Matrix of thin rectangular plate for non-dimensional coordinates.

Where $R = \frac{x}{a}$, $Q = \frac{y}{b}$, $dx = adR$, $dy = bdQ$ and $\alpha = \frac{b}{a}$

α is the aspect ratio while R and Q are non-dimensional axis parallel to x and y axis.

DETERMINATION OF DISPLACEMENT PROFILE FROM THE CHOSEN SHAPE FUNCTION OF THE PLATE.

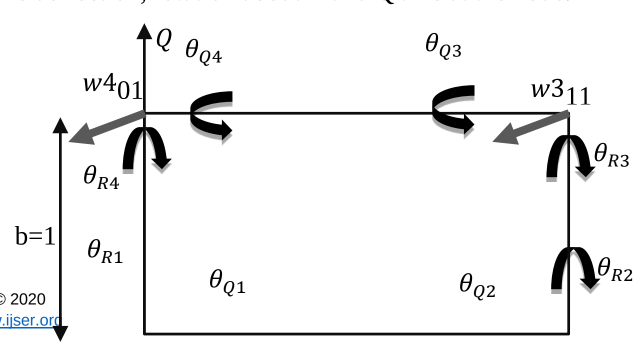
A chosen shape function of the plate N is expressed in Equation (11).

$$N = [1\quad R\quad Q\quad R2\quad RQ\quad Q2\quad R3\quad R2Q\quad RQ2\quad Q3\quad R3Q\quad RQ3] \qquad (11)$$

Ibearugbulem, [2].

Where R^2 and Q^2 denote R2 and Q2 respectively.

Figure 1 is a rectangular plate model that shows the 12 degrees of freedom of plate which will result to 12 by 12 matrix required for solution. a and b are the aspect ratio which is assumed to be 1. w, θR and θQ are deflection, rotation about R and Q axis at the node.



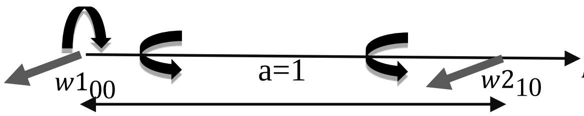


Figure 1. A thin plate with 12 degrees of freedom and nodal coordinates.

θR and θQ are obtained by differentiating Equation (11) with respect to R and Q. Hence the nodal displacement profile of the plate $[N_i]$ is expressed in Equation (12).

$$[N_i] = \begin{bmatrix} 1 & R & Q & R2 & RQ & Q2 & R3 & R2Q & RQ2 & Q3 & R3Q & RQ3 \\ 0 & 1 & 0 & 2R & Q & 0 & 3R2 & 2RQ & Q2 & 0 & 3R2Q & Q3 \\ 0 & 0 & 1 & 0 & R & 2Q & 0 & R2 & 2RQ & 3Q2 & R3 & 3RQ2 \end{bmatrix} \begin{matrix} N \\ \theta R \\ \theta Q \end{matrix} \quad (12)$$

In addition, using the nodal values corresponding to the coordinate of each of the node in the entire plate model, the square matrix of the nodal displacement profile of the thin plate is obtained in Equation (13).

$$[N_i] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 2 & 1 & 0 & 3 & 2 & 1 & 0 & 3 & 1 \\ 0 & 0 & 1 & 0 & 1 & 2 & 0 & 1 & 2 & 3 & 1 & 3 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 & 3 & 0 & 0 \end{bmatrix} \quad (13)$$

Applying the square matrix of the displacement profile and the plate shape function matrix into the general flexural element stiffness matrix equation, the fundamental natural frequency λ of the plate can be obtained.

FORMULATION OF THE RESONATING FREQUENCY EQUATION.

The general flexural element stiffness matrix equation of the plate c $w2\ 10$ ced to the following equations.

Let $K_R =$

$$[N_i^{-1}]^T \int_0^1 \int_0^1 [[N]''^T [N]'']_R \, dRdQ \,.\, [w_i][N_i^{-1}] \quad (14)$$

$K_{RQ} =$

$$2[N_i^{-1}]^T \int_0^1 \int_0^1 [[N]''^T [N]'']_{RQ} \, dRdQ \,.\, [w_i][N_i^{-1}] \quad (15)$$

$K_Q =$

$$[N_i^{-1}]^T \int_0^1 \int_0^1 [[N]''^T [N]'']_Q \, dRdQ \,.\, [w_i][N_i^{-1}] \quad (16)$$

$K_\lambda =$

$$-[N_i^{-1}]^T \int \int \big[[N]^T [N] \quad \big] \quad dRdQ \,.\, [w_i][N_i^{-1}] \quad (17)$$

Where K_λ is referred to as inertia matrix. Substituting Equation (14),(15),(16) and (17) into Equation (10) yields Equation (18).

$$\prod = \frac{D}{a^4}[K_R] + \frac{D}{a^4\alpha^2}[K_{RQ}] + \frac{D}{a^4\alpha^4}[K_Q] - m\lambda^2[K_\lambda] \quad (18)$$

The governing differential equation of plate in vibration stated in Equation (10) can be reduced to Equation (19).

$$\prod = \frac{D}{a^4}[K_R] + \frac{D}{a^4\alpha^2}[K_{RQ}] + \frac{D}{a^4\alpha^4}[K_Q] - m\lambda^2[K_\lambda] = 0 \quad (19)$$

$$m\lambda^2[K_\lambda] = \frac{D}{a^4}[K_R] + \frac{D}{a^4\alpha^2}[K_{RQ}] + \frac{D}{a^4\alpha^4}[K_Q] \quad (20)$$

Also, bringing out like terms in Equation (20) yields Equation (21).

$$m\lambda^2[K_\lambda] = \frac{D}{a^4}\left[K_R + \frac{1}{\alpha^2}K_{RQ} + \frac{1}{\alpha^4}K_Q\right] \quad (21)$$

$$\lambda^2 = \frac{D}{ma^4} \frac{\left[K_R + \frac{1}{\alpha^2}K_{RQ} + \frac{1}{\alpha^4}K_Q\right]}{[K_\lambda]} \quad (22)$$

$$\lambda = \frac{1}{a^2}\sqrt{\frac{D}{m} \cdot \frac{\left[K_R + \frac{1}{\alpha^2}K_{RQ} + \frac{1}{\alpha^4}K_Q\right]}{[K_\lambda]}} \quad (23)$$

Equation (23) is the resonating frequency equation of a plate in free vibration.

FINITE ELEMENT AND BOUNDARY CONDITION APPLICATION ON CCSS PLATE

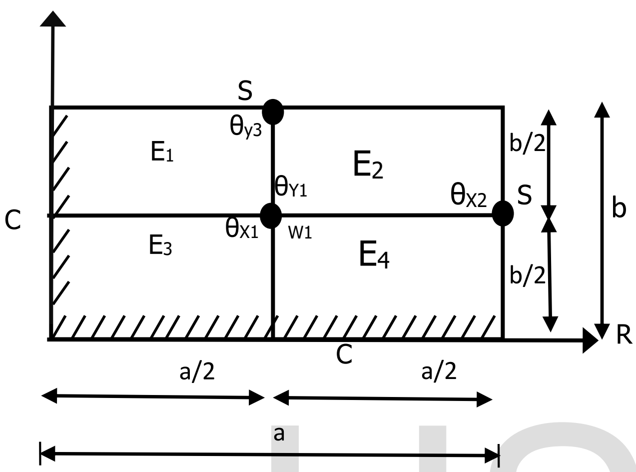


Figure 2. A CCSS thin plate with 5 degrees of freedom and 3 nodes.

The plate is divided into 4 finite elements as shown in Figure 2. It has a total number of 3 deformable nodes and 5 nodal displacements as listed.

w_1,θ_{x1},θ_{y1} at node 1, θ_{x2} at node 2, θ_{y3} at node 3.

BOUNDARY CONDITIONS FOR CCSS THIN PLATE.

At clamped edge R direction;

$w_R\ (R=0)=\ 0$ (24) $\theta X_R\ (R=0)=\ 0$ (26)

$\theta Y_R\ (R=0)=\ 0$ (28) $w_R\ (R=a)=\ 0$ (25)

$\theta X_R\ (R=a)=\ 0$ (27) $\theta Y_R\ (R=a)=\ 0$ (29)

At clamped edge Q direction;

$w_Q\ (Q=0)=\ 0$ (30) $\theta X_Q\ (Q=0)=\ 0$ (32)

$\theta Y_Q\ (Q=0)=\ 0$ (34) $w_Q\ (Q=b)=\ 0$ (31)

$\theta X_Q\ (Q=b)=\ 0$ (33) $\theta Y_Q\ (Q=b)=\ 0$ (35)

At simply supported edge R direction;

$w_R\ (R=0)=\ 0$ (36) $\theta X_R\ (R=0)=\ 0$ (38)

$\theta Y_R\ (R=0)=\ \theta y$ (40) $w_R\ (R=a)=\ 0$ (37)

$\theta X_R\ (R=a)=\ 0$ (39) $\theta Y_R\ (R=a)=\ \theta y$ (41)

At simply supported edge Q direction;

$w_Q\ (Q=0)=\ 0$ (42) $\theta X_Q\ (Q=0)=\ \theta x$ (44)

$\theta Y_Q\ (Q=0)=\ 0$ (46) $w_Q\ (Q=b)=\ 0$ (43)

$\theta X_Q\ (Q=b)=\ \theta x$ (45) $\theta Y_Q\ (Q=b)=\ 0$ (47)

The value of these displacements is obtained from the following individual stiffness matrixes

K_{R}, K_{RQ}, K_Q which is then summed to obtain the general stiffness matrix K and inertia matrix K_λ of the plate. The individual numerical stiffness matrix obtained from the global plate model in Figure 1 are shown in Equations (48),(49),(50) and (51).

$$K_R = \begin{bmatrix} 4 & 2 & 0 & -4 & 2 & 0 & -2 & 1 & 0 & 2 & 1 & 0 \\ 2 & 1.33333 & 0 & -2 & 0.66667 & 0 & -1 & 0.33333 & 0 & 1 & 0.66667 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4 & -2 & 0 & 4 & -2 & 0 & 2 & -1 & 0 & -2 & -1 & 0 \\ 2 & 0.66667 & 0 & -2 & 1.33333 & 0 & -1 & 0.66667 & 0 & 1 & 0.33333 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2 & -1 & 0 & 2 & -1 & 0 & 4 & -2 & 0 & -4 & -2 & 0 \\ 1 & 0.33333 & 0 & -1 & 0.66667 & 0 & -2 & 1.33333 & 0 & 2 & 0.66667 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & -2 & 1 & 0 & -4 & 2 & 0 & 4 & 2 & 0 \\ 1 & 0.66667 & 0 & -1 & 0.33333 & 0 & -2 & 0.66667 & 0 & 2 & 1.33333 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (48)$$

$$K_{RQ} = \begin{bmatrix} 2.8 & 0.2 & 0.2 & -2.8 & 0.2 & -0.2 & 2.8 & -0.2 & -0.2 & -2.8 & -0.2 & 0.2 \\ 0.2 & 0.2667 & 0 & -0.2 & -0.067 & 0 & 0.2 & 0.0667 & 0 & -0.2 & -0.267 & 0 \\ 0.2 & 0 & 0.2667 & -0.2 & 0 & -0.267 & 0.2 & 0 & 0.0667 & -0.2 & 0 & -0.067 \\ -2.8 & -0.2 & -0.2 & 2.8 & 0.2 & 0.2 & -2.8 & 0.2 & 0.2 & 2.8 & 0.2 & -0.2 \\ 0.2 & -0.067 & 0 & -0.2 & 0.2667 & 0 & 0.2 & -0.267 & 0 & -0.2 & 0.0667 & 0 \\ -0.2 & 0 & -0.267 & 0.2 & 0 & 0.2667 & -0.2 & 0 & -0.067 & 0.2 & 0 & 0.0667 \\ 2.8 & 0.2 & 0.2 & -2.8 & 0.2 & -0.2 & 2.8 & -0.2 & -0.2 & -2.8 & -0.2 & 0.2 \\ -0.2 & 0.0667 & 0 & 0.2 & -0.267 & 0 & -0.2 & 0.2667 & 0 & 0.2 & -0.067 & 0 \\ -0.2 & 0 & 0.0667 & 0.2 & 0 & -0.067 & -0.2 & 0 & 0.2667 & 0.2 & 0 & -0.267 \\ -2.8 & -0.2 & -0.2 & 2.8 & -0.2 & 0.2 & -2.8 & 0.2 & 0.2 & 2.8 & 0.2 & -0.2 \\ -0.2 & -0.267 & 0 & 0.2 & 0.0667 & 0 & -0.2 & -0.067 & 0 & 0.2 & 0.2667 & 0 \\ 0.2 & 0 & -0.067 & -0.2 & 0 & 0.0667 & 0.2 & 0 & -0.267 & -0.2 & 0 & 0.2667 \end{bmatrix} \quad (49)$$

ISSN 2229-5518

$$K_Q = \begin{bmatrix} 4 & 0 & 2 & 2 & 0 & 1 & -2 & 0 & 1 & -4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 1.3333 & 1 & 0 & 0.6667 & -1 & 0 & 0.3333 & -2 & 0 & 0.6667 \\ 2 & 0 & 1 & 4 & 0 & 2 & -4 & 0 & 2 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0.6667 & 2 & 0 & 1.3333 & -2 & 0 & 0.6667 & -1 & 0 & 0.3333 \\ -2 & 0 & -1 & -4 & 0 & -2 & 4 & 0 & -2 & 2 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0.3333 & 2 & 0 & 0.6667 & -2 & 0 & 1.3333 & -1 & 0 & 0.6667 \\ -4 & 0 & -2 & -2 & 0 & -1 & 2 & 0 & -1 & 4 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0.6667 & 1 & 0 & 0.3333 & -1 & 0 & 0.6667 & -2 & 0 & 1.3333 \end{bmatrix} \quad (50)$$

$$K_\lambda = \begin{bmatrix} 0.13706 & 0.01829 & 0.01829 & 0.04865 & -0.01087 & 0.00790 & 0.01563 & -0.00460 & -0.00460 & 0.04865 & 0.00790 & -0.01087 \\ 0.01829 & 0.00317 & 0.00250 & 0.01087 & -0.00238 & 0.00167 & 0.00460 & -0.00119 & -0.00111 & 0.00790 & 0.00159 & -0.00167 \\ 0.01829 & 0.00250 & 0.00317 & 0.00790 & -0.00167 & 0.00159 & 0.00460 & -0.00111 & -0.00119 & 0.01087 & 0.00167 & -0.00238 \\ 0.04865 & 0.01087 & 0.00790 & 0.13706 & -0.01829 & 0.01829 & 0.04865 & -0.00790 & -0.01087 & 0.01563 & 0.00460 & -0.00460 \\ -0.01087 & -0.00238 & -0.00167 & -0.01829 & 0.00317 & -0.00250 & -0.00790 & 0.00159 & 0.00167 & -0.00460 & -0.00119 & 0.00111 \\ 0.00790 & 0.00167 & 0.00159 & 0.01829 & -0.00250 & 0.00317 & 0.01087 & -0.00167 & -0.00238 & 0.00460 & 0.00111 & -0.00119 \\ 0.01563 & 0.00460 & 0.00460 & 0.04865 & -0.00790 & 0.01087 & 0.13706 & -0.01829 & -0.01829 & 0.04865 & 0.01087 & -0.00790 \\ -0.00460 & -0.00119 & -0.00111 & -0.00790 & 0.00159 & -0.00167 & -0.01829 & 0.00317 & 0.00250 & -0.01087 & -0.00238 & 0.00167 \\ -0.00460 & -0.00111 & -0.00119 & -0.01087 & 0.00167 & -0.00238 & -0.01829 & 0.00250 & 0.00317 & -0.00790 & -0.00167 & 0.00159 \\ 0.04865 & 0.00790 & 0.01087 & 0.01563 & -0.00460 & 0.00460 & 0.04865 & -0.01087 & -0.00790 & 0.13706 & 0.01829 & -0.01829 \\ 0.00790 & 0.00159 & 0.00167 & 0.00460 & -0.00119 & 0.00111 & 0.01087 & -0.00238 & -0.00167 & 0.01829 & 0.00317 & -0.00250 \\ -0.01087 & -0.00167 & -0.00238 & -0.00460 & 0.00111 & -0.00119 & -0.00790 & 0.00167 & 0.00159 & -0.01829 & -0.00250 & 0.00317 \end{bmatrix} \quad (51)$$

application of boundary conditions of CCSS plate listed from Equation (24) to Equation (47) yields a 5x5 stiffness matrix Equation for K and K_λ expressed in Equation (52) and Equation (53)

respectively. Where aspect ratio α=1

$$[K\ \ ] = \left[K_R + \frac{1}{\alpha^2}K_{RQ} + \frac{1}{\alpha^4}K_Q\right] = \begin{bmatrix} 43.2 & 0.4 & 0 & 4.4 & 4.4 \\ 0.4 & 6.4 & 0 & 1.1994 & 0 \\ 0 & 0 & 6.4 & 0 & 1.1994 \\ 0.4 & 1.1994 & 0 & 3.2 & 0 \\ 4.4 & 0 & 1.1994 & 0 & 3.2 \end{bmatrix} \quad (52)$$

$$K_\lambda = \begin{bmatrix} 0.54824 & 0 & 0 & -0.02174 & -0.02174 \\ 0 & 0.01268 & 0 & -0.00476 & 0 \\ 0 & 0 & 0.01268 & 0 & -0.00476 \\ -0.02174 & -0.00476 & 0 & 0.00634 & 0.00111 \\ -0.02174 & 0 & -0.00476 & 0.00111 & 0.00634 \end{bmatrix} \quad (53)$$

RESULTS AND DISCUSSION

The resonating natural frequency of CCSS plate of odd number grid size (i.e. plates divided into elements with central deflection as the point of consideration) was computed using Equation (23). For aspect ratio of 1 and grid size 1 the resonating natural frequency which is an eigenvalue problem was obtained as expressed in Equation (56). The remaining aspect ratios with 0.1 increment and the corresponding odd number grid size (n) ranging from 3.0 to 17 was computed. The grid size was to reflect the features of finite element as an approximate method of solution. Validation of the solution of this study was critically shown in Table 2. A comparison of this study was made with the solutions from other approximate methods like Njoku, [5], Sakata et al, [10] and Chakravarty,[1].

$$\lambda^2 = \frac{D}{ma^4}50.7068 \quad (54)$$

For one element, a = 0.5a

$$\lambda^2 = \frac{D}{m(0.5a)^4}50.7068 \quad (55)$$

$$\lambda = 28.4835\ \frac{1}{a^2}\sqrt{\frac{D}{m}} \quad (56)$$

Table 1. The natural frequency (λ) for different aspect ratio and grid size (n) for CCSS plates.

Aspect ratio α = b/a	Natural Frequency (λ) For Grid size n = 3	Natural Frequency (λ) For Grid size n = 5	Natural Frequency (λ) For Grid size n = 7	Natural Frequency (λ) For Grid size n = 9	Natural Frequency (λ) For Grid size n = 11	Natural Frequency (λ) For Grid size n = 13	Natural Frequency (λ) For Grid size n = 15	Natural Frequency (λ)For Grid size n = 17
1.0	26.20237	26.63259	26.80778	26.89367	26.94168	26.9711	26.99038	27.00369
1.1	23.95566	24.35108	24.51207	24.591	24.63512	24.66215	24.67987	24.6921
1.2	22.2895	22.66248	22.81424	22.88865	22.93024	22.95573	22.97244	22.98396
1.3	21.02467	21.38322	21.52896	21.60042	21.64037	21.66485	21.68089	21.69196
1.4	20.04478	20.39426	20.53606	20.60559	20.64446	20.66828	20.6839	20.69467
1.5	19.27194	19.61599	19.75524	19.82351	19.86168	19.88507	19.9004	19.91098
1.6	18.65261	18.99381	19.13144	19.19888	19.23695	19.2597	19.27485	19.2853
1.7	18.14914	18.48938	18.62601	18.69293	18.73034	18.75328	18.76831	18.77867
1.8	17.73452	18.07524	18.21131	18.2779	18.31513	18.33795	18.35291	18.36322
1.9	17.38896	17.73136	17.86717	17.93358	17.9707	17.99344	18.00835	18.01864
2.0	17.09779	17.44289	17.57867	17.64498	17.68203	17.70474	17.71962	17.72989

Table 2. Results of natural frequency (λ) for CCSS plate of present study and the results of previous studies with their percentage differences for different aspect ratio.

Aspect ratio α = b/a	Present Study (λ_1) For grid size n=17	Njoku, [5] (λ_2)	Sakata et al, [10] (λ_3)	Chakra-Verty [1] (λ_4)	Percentage Difference For (λ_2)&(λ_1)	Percentage Difference For (λ_3)&(λ_1)	Percentage Difference For (λ_4)&(λ_1)
1.0	27.00369	27.12846	26.867	27.055	0.4620	0.5062	0.1900
1.1	24.69210	24.80760	-	-	0.4677	-	-
1.2	22.98396	23.09477	-	-	0.4821	-	-
1.3	21.69196	21.80073	-	-	0.5014	-	-
1.4	20.69467	20.80073	-	-	0.5125	-	-
1.5	19.91098	20.01935	-	-	0.5443	-	-
1.6	19.28530	19.39417	-	-	0.5645	-	-
1.7	18.77867	18.88809	-	-	0.5826	-	-
1.8	18.36322	18.47313	-	-	0.5985	-	-
1.9	18.01864	18.12889	-	-	0.6118	-	-
2.0	17.72989	17.84034	17.770	-	0.6229	0.2262	-

Table 1. shows the resonating frequency (λ) of various odd number grid size (n). It was observed that the accuracy of the result increases with the increase in grid size. The increment in grid size shows the number of discretization the plate element undergoes which is one of the importance of finite element method. For aspect ratio of 1.2, the results of the natural frequency (λ) for grid size n=3 and n=17 are 22.2895 and 22.9724 respectively. Comparing the present solution as shown in table 2. with the solution given by Sakata et al, [10], Chakraverty,[1] and Njoku,[5] for an aspect ratio of 1.0 and grid size n=17, the resonating frequencies of these approximate methods are 26.867,27.055 and 27.12846 respectively. The corresponding percentage difference between the present study and other methods are 0.5062%, 0.1900% and 0.4614% respectively which shows how close their solutions are. The same comparison was done for aspect ratio 2.0 of the same grid size (n=17) on corresponding approximate solutions obtained by Sakata et al, [10], and Njoku,[5]. The percentage differences obtained were 0.2262% and 0.6229%. Hence, it can be deduced that the Formulated Flexural Element Stiffness Matrix in Finite Element Method for the selected shape function gave a solution close to other approximate methods. We can conclude that the present method is a good approximate method for analyzing plates subjected to free vibration.

REFERENCES

[1] Chakraverty, S. Vibration of Plates. *New York: CRC Press.* (2009).

[2] Ibearugbulem, Owus, M , Ezeh, J. C. & Ozuluonye, Chioma G., Pure Bending Analysis of SCSS and CSSS Platforms using *New Approach, International Journal of Science*,(2014).

[3] Ibearugbulem, O.M., E.I. Adah, D.O. Onwuka,C.P. Anyadiegwu. Computer Based Free Vibration Analysis of Isotropic Thin Rectangular Flat CCCC Plate. *International Journal of Engineering and Science, volume 5, issue 7, pp.38-41*,(2016).

[4] J. C. Ezeh, Ibearugbulem, M. Owus, H. E. Opara, and, O. A. Oguaghamba. Galerkin' Indirect Variational Method in Elastic Stability Analysis of All Edges Clamped Thin Rectangular Flat Plates, *International Journal of Research in Engineering and Technology EISSN: 2319-116 PISSN: 2321-7308, Volume: 03 Issue: 04, pp. 674 – 679, www.ijret.org*. (2014)

[5] Njoku K. O., J. C. Ezeh, O. M. Ibearugbulem, L. O. Ettu, L. Anyaogu. Free Vibration Analysis of Thin Rectangular Isotropic CCCC Plate Using Taylor Series Formulated Shape Function in Galerkin's Method. *Academic Research International, volume 4, issue 4, pp. 126-132*, (2013).

[6] Nikolas, T. Educational Examples in Structural Acoustics Using the Finite Element Method. Pp. 29-30, (2014).

[7] Oba, E.F., O.M. Ibearugbulem, C.N. Ibearugbulem, K.O. Njoku and Anyogu, L. Use of Variational Calculus to Evolve Third Order Functional for Continuum Analysis. *International Journal of Innovative Research and Development, volume 16, issue 4, pp.52-58*, (2017).

[8] Okafor F.O. and Udeh O.T. Direct Method of Analysis of an Isotropic Rectangular Plate Using Characteristic Orthogonal Polynomials. *Nigerian Journal of Technology, volume 34, No.2, pp. 232-239*. (2015)

[9] Ramu, I and S.C. Monhanty. Study on Free Vibration Analysis of Rectangular Plates Structures Using Finite Element Method. *Research Scholar, Department of Mechanical Engineering, Procedia Engineering, 38, 2758-2766. (2012).*

[10] Sakata,T., Takahashi, K. and Bhat, R.B. Natural Frequencies of Orthotropic Rectangular Plates Obtained by Iterative Reduction of the Partial Differential Equation. *Journal of sound Vibration, 18, pp.89-101*. (1996).

[11] Supun J. and Seyed M.H. A Dynamic Coefficient Matrix Method for Free Vibration of Thin Rectangular Plates. *Journal of Shock and Vibration, volume 2018, ID 1071830*, (2018).